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Modeling of Turbine Engine Axial-Flow Compressor and Turbine Characteristics

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This paper describes the concept of modeling the turbine engine axial-flow compressor and turbine characteristics. The conception is based on the use of two variables to the approximation of these characteristics' curves. It gives the possibility of modeling the whole area of their action and provides the utmost accuracy of their representation. The conception is original because it presents changes of a compressor and turbine's parameters of work in the unconventional coordinate system, as suggested by the authors. The accuracy of the conception of modeling has been tested on the exemplary characteristics of a compressor and turbine in a turbine engine. Research is carried out in the following spheres: 1) evaluation of the degradation of engine components' influence on the turbomachinery working line, and 2) evaluation of the inefficiency of the fuel supply system's influence on the change of the thermodynamic cycle's parameters in the stationary and transient states. This research will also include a nonlinear digital model of a turbine engine, and the conception of modeling characteristics of rotating components, and the mass accumulation in the subassemblies of the engine.

Nomenclature

$\dot{m}$ = mass flow
$\dot{m}_c$ = corrected mass flow, $\dot{m}\sqrt{(\Theta)}/\delta$
$\bar{\dot{m}}_c$ = $\dot{m}_c/\dot{m}_{\mathrm{REF}}$
N = rotational speed, rpm
N_c = corrected rotational speed, $N/\sqrt{\Theta}$
$\bar{N}_c$ = N_c/N_{REF}
Z = coefficient of relative stability margin of the compressor
δ = dimensionless pressure
η_C = compressor adiabatic efficiency, total–total
η_T = turbine adiabatic efficiency, total–total
Θ = dimensionless temperature
Π_C = compressor total pressure ratio
$\bar{\Pi}_C$ = $\Pi_C/\Pi_{C\mathrm{REF}}$
Π_T = turbine total pressure ratio
$\bar{\Pi}_T$ = $\Pi_T/\Pi_{T\mathrm{REF}}$

Subscripts

REF = reference point
sl = surge line
2 = compressor entry
4 = turbine entry

I. Introduction

AN aircraft gas-turbine engine represents a complex system where the overall engine performance and components matching depends on the flight parameters, design, and operational conditions. The analysis of the influence of several of these parameters can be explored numerically, but the condition of the utmost accuracy of individual components representation must be fulfilled. However, the majority of problems in developing an adequate gas-turbine simulation model are related to inaccuracies in the prediction of the component characteristics, particularly for the compressor and turbine (see Figs. 1 and 2). A review of the available simulation models presented in the literature indicate that gas-turbine simulation models usually require the following formulation of the functional relations that exist between characteristic variables of interest:

$$\begin{aligned} \dot{m}_{2c} &= \dot{m}_{2c}(N_{2c}, \Pi_C), \qquad & \eta_C &= \eta_C(N_{2c}, \Pi_C) \\ \Pi_T &= \Pi_T(N_{4c}, \dot{m}_{4c}), \qquad & \eta_T &= \eta_T(N_{4c}, \Pi_T) \end{aligned} \tag{1}$$

Focusing attention on the modeling problems with compressor and turbine characteristics, let us mention several potential sources of troubles. These are as follows:

1) Determination of the shape of characteristics (see Figs. 1 and 2).
2) The nonuniqueness problem.
3) The ill-conditioning problem, where some small changes in the variable of one coordinate produce large changes in the other coordinate variables.
4) The large variation encountered in the variables if the full envelope of the compressor or turbine characteristics is desired.
5) The differences in the order of magnitude of variables (scaling problem).

A review of the published papers on this subject shows that there is still a need for a general solution that can be used to model the compressor and turbine performance map. Dobryanskiĭ and Martyanova[1] used look-up tables to store the characteristics and linear or Lagrangian interpolation technique to determine the values of the performance parameters for an arbitrarily selected point on the performance map. A similar technique was also applied by Ismail and Bhinder.[2] The rational functions and an ellipse approximation introduced by Hormouziadis and Herbig[3] to describe the speed curves and efficiency contours on a compressor performance map also cover the various aspects of modeling the compressor characteristics. To obtain improved model outputs, El-Gammal[4] developed the criteria and algorithm for a compressor characteristic's linear model. This method allows one to objectively choose the most adequate model according to known performance data. Recent improvements in compressor and turbine map representations, by the use of analytical functions, have led to the use of nonlinear models. This kind of map description presents the possibility to successfully model maps of different design types. A method of this type has been used by Sieros et al.[5]

The aim of this study is to describe a concept of modeling the compressor and turbine characteristics that has been developed specifically for the digital simulation of the transient-state operation of gas-turbine engines. The main innovation is the adaptation of the

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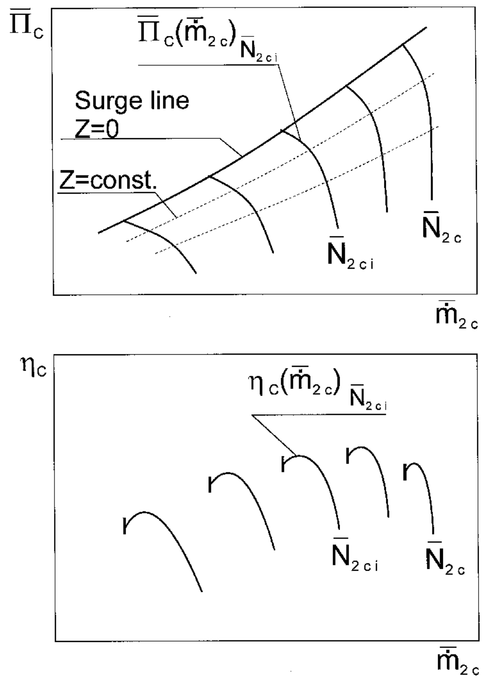

Fig. 1 Typical compressor performance map.

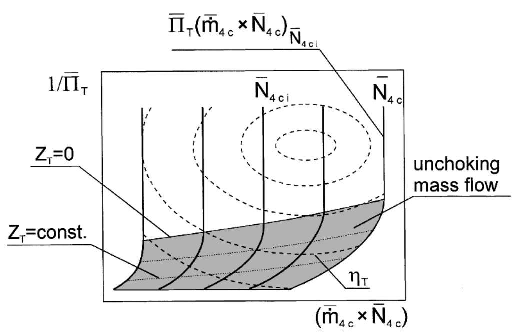

Fig. 2 Typical turbine performance map.

functions of two variables and original data rescaling. In addition, this concept can be used to represent either a part or full envelope of characteristics, without the normally encountered limits. A method of this type has already been employed by Orkisz and Stawarz,[6] but only for a compressor map. The further advancement of that modeling concept for both compressor and turbine maps has also been discussed in Ref. 7.

II. Modeling of Compressor Performance Map

The compressor performance map, available in a traditional plane of reference, can be plotted by means of appropriate data rescaling in a coordinate system of $(\bar N_{2c}, Z, \bar m_{2c})$ and $(\bar N_{2c}, Z, \eta_C)$, as shown in Fig. 3. For the following new relations it is possible to find the function of best approximation, without the limits as previously introduced:

$$\bar m_{2c} = \bar m_{2c}(\bar N_{2c}, Z), \qquad \eta_C = \eta_C(\bar N_{2c}, Z) \tag{2}$$

Before finding the analytical closed forms of relations [Eq. (2)], it is necessary to store the compressor characteristic as tabulated data points. The corrected mass flow $\bar m_{2ci}$ and compressor efficiency η_{Cij}, with assumed values of $\bar N_{2ci}$ and Z_j parameters, can be obtained by solving the following equations numerically:

$$\left(\frac{\bar\Pi_{C\mathrm{sl}}}{\bar m_{2c\mathrm{sl}}}\frac{\bar m_{2c}}{\bar\Pi_C(\bar m_{2c})}\right)_{\bar N_{2ci}} - 1 = Z_j, \qquad \eta_{Cij} = \eta_C(\bar m_{2cij})_{\bar N_{2ci}} \tag{3}$$

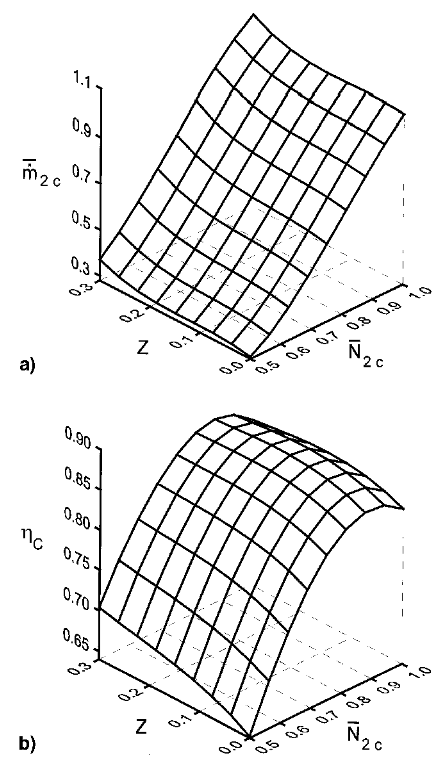

Fig. 3 Graphical representation of example interrelation: a) $\bar m_{2c} = \bar m_{2c}(\bar N_{2c}, Z)$ and b) $\eta_C = \eta_C(\bar N_{2c}, Z)$ for compressor.

where $\bar\Pi_C(\bar m_{2c})$ and $\eta_C(\bar m_{2cij})$ are polynomials of the best approximation of the ith speedlines (see Fig. 1). Then, assuming the form of relations [Eq. (2)], in specific classes of functions of two variables, e.g., in second-degree polynomials:

$$\bar m_{2c} = \bar m_{2c}(\bar N_{2c}, Z) = A_0 + A_1\bar N_{2c} + A_2\bar N_{2c}^2 + A_3\bar N_{2c}Z + A_4Z + A_5Z^2$$

$$\eta_C = \eta_C(\bar N_{2c}, Z) = B_0 + B_1\bar N_{2c} + B_2\bar N_{2c}^2 + B_3\bar N_{2c}Z + B_4Z + B_5Z^2 \tag{4}$$

the coefficients A_i and B_i of the fitting functions can be determined using a least-squares method. Additionally, the surge line should be expressed as a function of corrected rotational speed in the following form:

$$\bar m_{2c\,\mathrm{sl}} = \bar m_{2c\,\mathrm{sl}}(\bar N_{2c}), \qquad \bar\Pi_{C\,\mathrm{sl}} = \bar\Pi_{C\,\mathrm{sl}}(\bar N_{2c}) \tag{5}$$

Having completed the previous steps, an analytical representation of the compressor performance is described by the algorithm in Fig. 4. The iterative solution scheme has been tailored to gas-turbine simulation applications so that it will converge on the solution for the wide range of input values.

III. Modeling of Turbine Performance Map

As in the case of the compressor, a similar technique can also be applied to describe the behavior of the turbine performance data. The turbine performance characteristics are usually plotted as shown in Fig. 2, after several manipulations that can be presented in another coordinate system of $(\bar m_{4c} \times \bar N_{4c}, Z_T, \bar\Pi_T)$ and $(\bar N_{4c}, \bar\Pi_T, \eta_T)$. The typical trends of the new, rescaled characteristic is shown in Fig. 5. The Z_T parameter is defined as

$$Z_T = \frac{\bar\Pi_{Tzt}}{(\bar m_{4c} \times \bar N_{4c})_{zt}}\frac{\bar m_{4c} \times \bar N_{4c}}{\bar\Pi_T} - 1 \tag{6}$$

where zt is the index referring to the $Z_T = 0$ line data values (see Fig. 2).

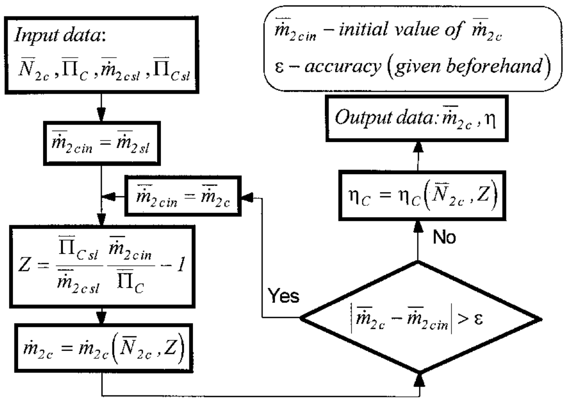

Fig. 4 Flow diagram for model of compressor performance map.

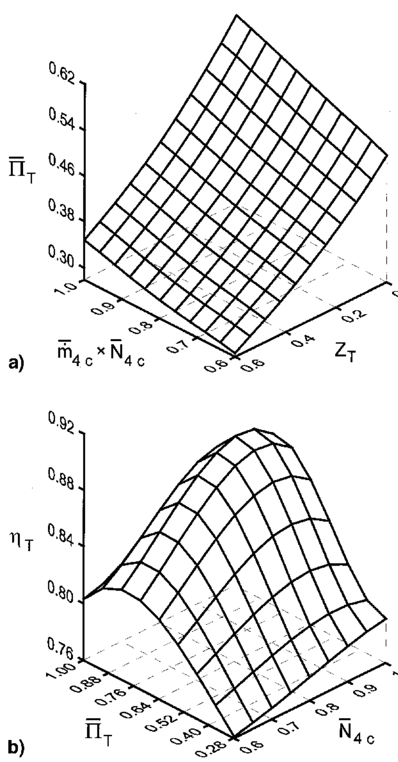

Fig. 5 Graphical representation of example interrelation: a) $\bar{\Pi}_T = \bar{\Pi}_T(\bar{\dot{m}}_{4c} \times \bar{N}_{4c}, Z_T)$ and b) $\eta_T = \eta_T(\bar{N}_{4c}, \bar{\Pi}_T)$ for turbine.

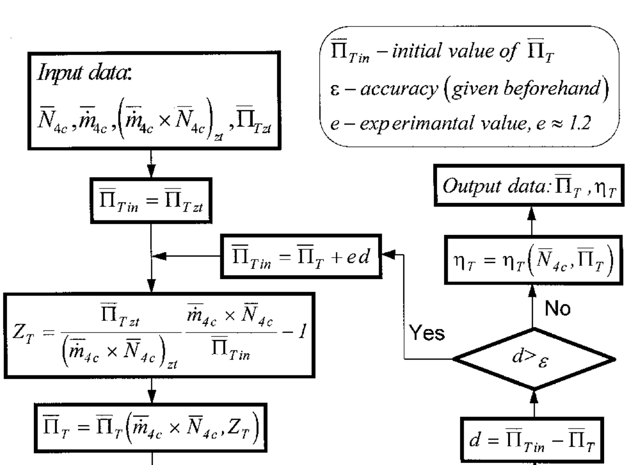

Fig. 6 Flow diagram for model of turbine performance map.

The following suitable fitting functions of the given relations existing between turbine variables can be found in the previously introduced manner:

$$\bar{\Pi}_T = \bar{\Pi}_T(\bar{\dot{m}}_{4c} \times \bar{N}_{4c}, Z_T) \tag{7}$$

$$\eta_T = \eta_T(\bar{N}_{4c}, \bar{\Pi}_T) \tag{8}$$

Additionally, the $Z_T = 0$ line (see Fig. 2) should be expressed as a function of corrected rotational speed in the following form:

$$\bar{\dot{m}}_{2czt} = \bar{\dot{m}}_{2czt}(\bar{N}_{2c}), \qquad \bar{\Pi}_{Czt} = \bar{\Pi}_{Czt}(\bar{N}_{2c}) \tag{9}$$

Substituting Eqs. (7) and (8) into the algorithm that is particularly designed for that purpose, we can finally establish an analytical description of the turbine performance map. Figure 6 shows the logic of an iteration solution sequence. If the experimental value e is greater than 1, then the iterative process is convergent.

It should be noted that Eq. (7) cannot cover the choked region of turbine characteristics because the mass flow through the turbine operating at a choked condition is a function of minimum cross sections of turbine and exhaust nozzles.

IV. Results

To analyze the predictability of component behavior by using the described modeling methodology, various compressor and turbine characteristics have been considered. For all of the test cases, the satisfactory level of characteristics' prediction is guaranteed by the functions of best approximation in the form of second- or

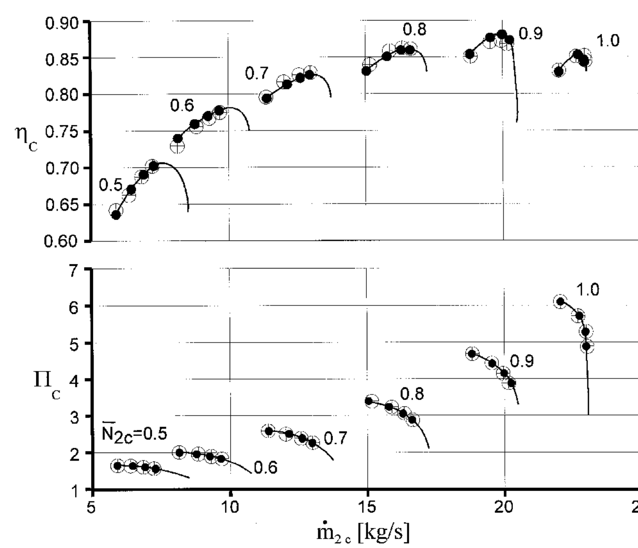

Fig. 7 Original (●) and the corresponding predicted (⊕) corrected mass flow and compressor efficiency.

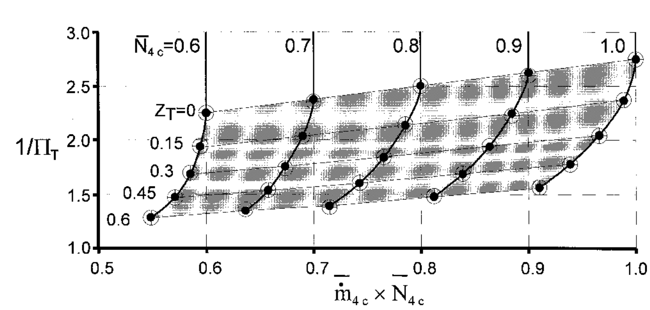

Fig. 8 Original (●) and the corresponding predicted (⊕) turbine expansion ratio.

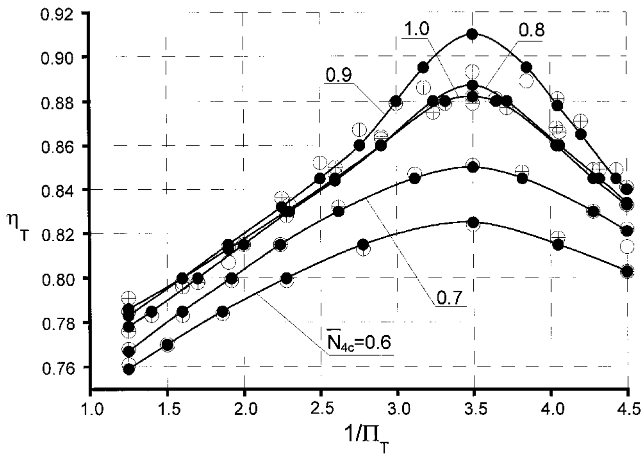

Fig. 9 Original (●) and the corresponding predicted (⊕) turbine efficiency.

third-degree polynomials, where the optimum form of these functions is obtained through the trial-and-error process. The relative error was between 1.0 and 2.0%.

In the case of the compressor characteristics, the comparison of the predicted data points with the corresponding original points for one of the examined compressors results in a maximum error of <1.5% for corrected mass flow and efficiency. Figure 7 illustrates the original and predicted corrected mass flow and efficiency values of this map.

For the examined turbine map, the predicted data points compared with the corresponding original points give the maximum error <2% for the expansion ratio and efficiency. Figure 8 illustrates the original and predicted expansion ratio, whereas Fig. 9 shows the original and predicted efficiency values of this characteristic.

V. Conclusions

1) This paper describes a concept that has been used to model compressor and turbine performance maps.

2) The proposed modeling technique has been used for modeling a variety of compressor and turbine maps, and it is possible to find the functions of two variables that accurately represents a component map.

3) The presented modeling methodology is easy to use and enables one to model either a part or the entire area of characteristics.

4) The computational results have shown that the presented algorithm is capable of modeling various compressor and turbine characteristics, with a high degree of agreement between these results and the experimental data. Thus, it is useful for a complex simulation of the transient state operation of gas-turbine engines.

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